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PROPAGATION OF RAYLEIGH WAVES IN TRANSVERSELY ISOTROPIC GENERALIZED THERMOELASTIC DIFFUSION

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UDC 536.21

This paper is devoted to the study of propagation of Rayleigh waves in a homogeneous, transversely isotropic, thermoelastic diffusive half-space that is subjected to stress-free, thermally insulated/isothermal, and chemical potential boundary conditions in the context of the generalized theory of thermoelastic diffusion. The Lord and Shulman theory, where thermal and thermomechanical relaxation as well as diffusion relaxation are governed by two different time constants, is selected. Secular equations for surface wave propagation in the considered media are derived. The amplitudes of surface displacements, temperature change, and concentration are computed. The paths of the surface particles are determined. Transverse isotropy and diffusion effects on the phase velocity, group velocity, and attenuation coefficient are presented graphically.

Keywords: *Wave propagation; transversely isotropic solid; generalized thermoelastic diffusion; phase velocity; group velocity; attenuation coefficient*

Introduction. During the last three decades, nonclassical theories of thermoelasticity, so-called "generalized thermoelasticity," have been developed in order to remove the paradox of the physically impossible phenomenon of infinite velocity of propagation of thermal signals in the conventional coupled thermoelasticity. The Lord–Shulman theory [1] and Green–Lindsay theory [2] are important generalized theories of thermoelasticity that become a center of interest of recent researches in this area. In the Lord–Shulman theory, the flux rate term with one relaxation time is incorporated into the Fourier law of heat conduction and a generalized theory admitting finite speed for thermal signals is formulated. The Green–Lindsay theory [2], called the theory of temperature rate-dependent thermoelasticity with two constants that act as two relaxation times, does not violate the classical Fourier law of heat conduction when the body under consideration has a center of symmetry. The Lord and Shulman theory [1] of generalized thermoelasticity was further extended to homogeneous anisotropic heat conducting materials by Dhaliwal and Sherief [3]. All these theories predict a finite speed of heat propagation. Chanderashekhariah [4] refers to this wave-like thermal disturbance as "second sound." A survey article of various representative theories in the range of generalized thermoelasticity was suggested by Hetnarski and Ignaczak [5].

Spontaneous movement of particles from the high concentration region to the low concentration one is defined as diffusion, and it occurs due to a concentration gradient. An example of diffusion is heat or momentum transport. The net flux of a transported quantity (atoms, energy, or electrons) is equal to a physical property (diffusivity, thermal conductivity, or electrical conductivity) multiplied by a corresponding gradient (of concentration, thermal, or electrical field). Nowadays, there is a great deal of interest in the study of this phenomenon due to its applications in the geophysics and electronic industry. Technologies based on diffusive waves have improved biomedical diagnostics and fabrication of optical and electronic devices. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amounts into the semiconductor substance. In particular, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors, as well as the source/drain regions and dope polysilicon gates in metal–oxide–semiconductor transistors. Thermodiffusion utilizes the heat transfer across a thin liquid or gas to accomplish isotope separation. Today, thermodiffusion remains a practical process to separate isotopes of noble gases (e.g., xenon) and other light isotopes (e.g., carbon) for research purposes. In most of the applications, the concentration is calculated

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using Fick's law. This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the temperature effect on this interaction. However, there is a certain degree of coupling with temperature and temperature gradient as temperature accelerates the diffusion process. The thermodiffusion in elastic solids is due to coupling of the fields of temperature, mass diffusion, and strain in addition to the heat and mass exchange with the environment.

Nowacki [6–9] developed the theory of thermoelastic diffusion by using a coupled thermoelastic model. Dudziak and Kowalski [10] and Olesiak and Pyryev [11], respectively, discussed the theory of thermodiffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer. They studied the influence of cross effects arising from the coupling of the fields of temperature, mass diffusion, and strain due to which the thermal excitation results in additional mass concentration and generates additional temperature fields.

Sherief et al. [12] developed the generalized theory of thermoelastic diffusion with one relaxation time, which allows the finite speed of propagation of waves. Sherief and Saleh [13] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh [14, 15] discussed the phenomenon of wave reflection from the free surface of an elastic solid with generalized thermodiffusion. Aouadi [16–20] investigated different types of thermoelastic diffusion problems. Sharma [21, 22] discussed the plane harmonic generalized thermoelastic diffusive waves and elasto-thermodiffusive surface waves in heat-conducting solids.

In spite of these studies, no attempt has been made to study the effect of diffusion on wave propagation in transversely isotropic and isotropic generalized thermoelastic diffusive half-spaces. Keeping in view the above applications of thermodiffusion processes, in what follows the propagation of Rayleigh waves in a homogeneous, transversely isotropic, generalized thermoelastic diffusive half-space will be investigated. The phase velocity, group velocity, and attenuation coefficient of wave propagation will be obtained from the secular equations.

Basic Equations. The basic governing equations for a homogeneous anisotropic, generalized thermoelastic diffusive solid in the absence of body forces, heat sources, and diffusive mass sources are given by

$$\sigma_{ij} = c_{ijkm} e_{km} + a_{ij} T + b_{ij} C \,, \tag{1}$$

$$q_i + \tau_0 \dot{q}_i = -K_{ij} T_{,j} \,, \tag{2}$$

$$\eta_i + \tau^0 \dot{\eta}_i = -\alpha^*_{ij} P_{,j} \,, \tag{3}$$

$$\rho S T_0 = \rho C_E T + a T_0 C - a_{ij} e_{ij} T_0 \,, \tag{4}$$

$$P = b_{km} e_{km} + bC - aT \,. \tag{5}$$

The equation of motion, entropy equation, and the equation of conservation of mass are, respectively,

$$\sigma_{ij,j} + \rho F_i = \rho \ddot{u}_i \,, \tag{6}$$

$$q_{i,i} + \rho T_0 \dot{S} - \rho M + P \eta_{i,i} = 0 \,, \tag{7}$$

$$\eta_{i,i} = \dot{C} + \rho N \,. \tag{8}$$

Here, $a_{ij} = a_{ji}$, $c_{ijkm} = c_{kmij} = c_{jikm} = c_{ijmk}$, $b_{ij} = b_{ji}$, $e_{ij} = e_{ji}$, $K_{ij} = K_{ji}$, $\alpha^*_{ij} = \alpha^*_{ji}$, $\sigma_{ij} = \sigma_{ji}$ and the comma in the subscripts denotes differentiation with respect to a spatial coordinate.

Substituting Eqs. (1)–(5) into Eqs. (6)–(8) without body, heat, and diffusive mass forces, we obtain the following equations of motion, heat conduction, and mass diffusion:

$$c_{ijkm}e_{km,j} + a_{ij}T_{,j} + b_{ij}C_{,j} = \rho\ddot{u}_i \,, \tag{9}$$

$$\rho C_E\,(\dot{T} + \tau_0\ddot{T}) + aT_0\,(\dot{C} + \tau_0\ddot{C}) - a_{ij}T_0\,(\dot{e}_{ij} + \tau_0\ddot{e}_{ij}) = K_{ij}T_{,ij} \,, \tag{10}$$

$$-\,\overset{*}{\alpha}_{ij}b_{km}e_{km,ij} - \overset{*}{\alpha}_{ij}bC_{,ij} + \overset{*}{\alpha}_{ij}aT_{,ij} = -\,(\dot{C} + \tau^0\ddot{C}) \,. \tag{11}$$

Applying the transformation

$$x_1' = x_1 \cos\phi + x_2 \sin\phi \,, \quad x_2' = -\,x_1 \sin\phi + x_2 \cos\phi \,, \quad x_3' = x_3 \,, \tag{12}$$

where ϕ is the angle of rotation in the x_1x_2 plane, from Eqs. (9)–(11) we obtain the following basic equations for a homogeneous, transversely isotropic, coupled thermodifffusive elastic solid:

$$c_{11}u_{1,11} + c_{12}u_{2,21} + c_{13}u_{3,31} + c_{66}\,(u_{1,22} + u_{2,12}) + c_{44}\,(u_{1,33} + u_{3,13}) - a_1T_{,1} - b_1C_{,1} = \rho\ddot{u}_1 \,, \tag{13}$$

$$c_{66}\,(u_{1,21} + u_{2,11}) + c_{12}u_{1,12} + c_{11}u_{2,22} + c_{44}u_{2,33} + (c_{13} + c_{44})\,u_{3,32} - a_1T_{,2} - b_1C_{,2} = \rho\ddot{u}_2 \,, \tag{14}$$

$$(c_{13} + c_{44})\,(u_{1,13} + u_{2,23}) + c_{44}\,(u_{3,11} + u_{3,22}) + c_{33}u_{3,33} - a_3T_{,3} - b_3C_{,3} = \rho\ddot{u}_3 \,, \tag{15}$$

$$\begin{gathered}\rho C_E\,(\dot{T} + \tau_0\ddot{T}) + aT_0\,(\dot{C} + \tau_0\ddot{C}) + [a_1\,(\dot{u}_{1,1} + \dot{u}_{2,2} + \tau_0\,(\ddot{u}_{1,1} + \ddot{u}_{2,2}))\\ +\, a_3\,(\dot{u}_{3,3} + \tau_0\ddot{u}_{3,3})]\,T_0 = K_1\,(T_{,11} + T_{,22}) + K_3T_{,33} \,,\end{gathered} \tag{16}$$

$$\begin{gathered}\overset{*}{\alpha}_1\,[b_1\,(u_{1,111} + u_{2,222} + u_{2,211} + u_{1,122}) + b_3\,(u_{3,311} + u_{3,322})] + \overset{*}{\alpha}_3\,[b_1\,(u_{1,133} + u_{2,233}) + b_3u_{3,333}]\\ -\,\overset{*}{\alpha}_1b\,(C_{,11} + C_{,22}) - \overset{*}{\alpha}_3bC_{,33} + \overset{*}{\alpha}_1a\,(T_{,11} + T_{,22}) + \overset{*}{\alpha}_3aT_{,33} = -\,(\dot{C} + \tau^0\ddot{C}) \,,\end{gathered} \tag{17}$$

where

$$\begin{gathered}a_{ij} = -\,a_i\delta_{ij} \,, \quad b_{ij} = -\,b_i\delta_{ij} \,, \quad \overset{*}{\alpha}_{ij} = \overset{*}{\alpha}_i\delta_{ij} \,, \quad K_{ij} = K_i\delta_{ij} \,,\\ a_1 = (c_{11} + c_{12})\,\alpha_1 + c_{13}\alpha_3 \,, \quad a_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3 \,,\\ b_1 = (c_{11} + c_{12})\,\alpha_{1\mathrm{c}} + c_{13}\alpha_{3\mathrm{c}} \,, \quad b_3 = 2c_{13}\alpha_{1\mathrm{c}} + c_{33}\alpha_{3\mathrm{c}} \,,\\ c_{66} = (c_{11} - c_{12})/2 \,.\end{gathered} \tag{18}$$

Here α_i and α_{ic} are the coefficients of linear thermal expansion and diffusion expansion, respectively. In Eqs. (13)–(17) we used for brevity the figures 1, 2, 3, 4, 5, and 6 instead of the indices 11, 22, 33, 23, 13, and 12, respectively, to relate c_{ijkm} to c_{ln} $(i, j, k, m = 1, 2, 3$ and $l, n = 1, 2, 3, 4, 5, 6)$.

Formulation of the Problem. We consider a homogeneous, transversely isotropic, generalized thermodiffusive elastic half-space that is kept initially at a uniform temperature T_0. The origin of the coordinate system (x_1, x_2, x_3) is taken at any point on the plane horizontal surface. We take the x_3 axis along the axis of material symmetry with the vertical direction downward into the half-space (which is thus represented by $x_3 \geq 0$). The surface $x_3 = 0$ is subjected to traction-free, thermally insulated or isothermal, and chemical potential boundary conditions. We chose the x_1 axis in the direction of wave propagation, so that all particles on a line parallel to the x_2 axis are equally displaced. Therefore, all the field quantities will be independent of the coordinate x_2. Then, for a two-dimensional problem

$$\mathbf{u}(x_1, x_3, t) = (u_1, 0, u_3), \quad T = T(x_1, x_3, t), \quad C = C(x_1, x_3, t) \tag{19}$$

are the displacement vector, temperature change, and concentration.

We define the dimensionless quantities

$$x_i' = \frac{\omega_1^* x_i}{v_1}, \quad t' = \omega_1^* t, \quad u_i' = \frac{\omega_1^* u_i}{v_1}, \quad T' = \frac{a_1 T}{\rho v_1^2}, \quad C' = \frac{b_1 C}{\rho v_1^2}, \quad P' = \frac{P}{b_1},$$

$$\tau_0' = \omega_1^* \tau_0, \quad \tau^{0'} = \omega_1^* \tau^0, \quad h' = \frac{v_1 h}{\omega_1^*}, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{a_1 T_0}, \quad v_1^2 = \frac{c_{11}}{\rho}, \quad \omega_1^* = \frac{\rho C_E v_1^2}{K_1}. \tag{20}$$

Here ω_1^* is the characteristic frequency of the medium and v_1 is the longitudinal wave velocity. Introducing these quantities into Eqs. (13)–(17), omitting the primes, using Eq. (19), and assuming the solutions in the form

$$(u_1, u_{3,} T, C) = (1, W, S, R) \exp\,[i\xi\,(x_1 + m x_3 - ct)], \tag{21}$$

we obtain the following polynomial characteristic equation:

$$m^8 + A^* m^6 + B^* m^4 + C^* m^2 + D^* = 0. \tag{22}$$

The coefficients A^*, B^*, C^*, and D^* are given in the Appendix. The characteristic equation (22) is biquadratic in m^2 and hence possesses four roots $m_p^2, p = 1, 2, 3, 4$, We are interested in surface waves only, consequently it is essential that the motion is confined to the free surface $x^3 = 0$ of the half-space, so that the characteristic roots m_p^2 must satisfy the radiation condition $\mathrm{Re}(m_p) \geq 0$. Then the formal expressions for the displacements, temperature change, and concentration can be written as

$$(u_1, u_3, T, C) = \sum_{p=1}^{4} (1, n_{1p}, n_{2p}, n_{3p})\, A_p \exp\,[i\xi\,(x_1 + i m_p x_3 - ct)], \tag{23}$$

where A_p are arbitrary constants. The coupling constants n_{1p}, n_{2p} and n_{3p}, are given in the Appendix.

Boundary Conditions. The nondimensional boundary conditions at the surface x_3 are given by *mechanical conditions* (stress-free surface)

$$\sigma_{33} = (\delta_2 - \delta_1)\, u_{1,1} + \delta_4 u_{3,3} - (\varepsilon_1 T + \varepsilon_2 C) = 0,$$

$$\sigma_{31} = \delta_1\,(u_{3,1} + u_{1,3}) = 0; \tag{24}$$

thermal conditions

$$T_{,3} + hT = 0, \tag{25}$$

where h is the surface heat transfer coefficient ($h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ refers to isothermal surfaces);

chemical potential condition

$$P = u_{1,1} + p_2 u_{3,3} - n_2 C + n_1 T = 0\,, \tag{26}$$

where

$$n_1 = \frac{a c_{11}}{a_1 b_1}\,, \quad n_2 = \frac{b c_{11}}{b_1^2}\,.$$

Expressions for δ_i and ε_i are presented in the Appendix.

Derivation of the Secular Equations. Substituting the expressions for u_1, u_3, T, and C from Eq. (23) into the boundary conditions (24)–(26), we obtain the secular equation

$$P_{41} D_1 - P_{42} D_2 + P_{43} D_3 - P_{44} D_4 = 0\,, \tag{27}$$

where

$$D_1 = P_{12}\,(P_{23}P_{34} - P_{33}P_{24}) - P_{13}\,(P_{22}P_{34} - P_{32}P_{24}) + P_{14}\,(P_{22}P_{33} - P_{32}P_{23})\,,$$

$$D_2 = P_{11}\,(P_{23}P_{34} - P_{33}P_{24}) - P_{13}\,(P_{21}P_{34} - P_{31}P_{24}) + P_{14}\,(P_{21}P_{33} - P_{31}P_{23})\,,$$

$$D_3 = P_{11}\,(P_{22}P_{34} - P_{32}P_{24}) - P_{12}\,(P_{21}P_{34} - P_{31}P_{24}) + P_{14}\,(P_{21}P_{32} - P_{31}P_{22})\,,$$

$$D_4 = P_{11}\,(P_{22}P_{33} - P_{32}P_{23}) - P_{12}\,(P_{21}P_{33} - P_{31}P_{23}) + P_{13}\,(P_{21}P_{32} - P_{31}P_{22})\,,$$

$$P_{1p} = \delta_2 - \delta_1 + i\delta_4 m_p n_{1p} + \frac{i\varepsilon_1 n_{2p}}{\xi} + \frac{i\varepsilon_2 n_{3p}}{\xi}\,, \quad P_{2p} = n_{1p} + i m_p\,,$$

$$P_{3p} = 1 + i\varepsilon_2 m_p n_{1p} + \frac{i n_2 n_{3p}}{\xi} - \frac{i n_1 n_{2p}}{\xi}\,, \quad P_{4p} = (i m_p + h)\, n_{2p}\,, \quad p = 1, 2, 3, 4\,.$$

For thermally insulated and isothermal thermoelastic diffusion half-spaces, the secular equation (27) becomes

$$m_1 n_{21} D_1 - m_2 n_{22} D_2 + m_3 n_{23} D_3 - m_4 n_{24} D_4 = 0\,, \tag{28}$$

$$n_{21} D_1 - n_{22} D_2 + n_{23} D_3 - n_{24} D_4 = 0\,. \tag{29}$$

Equations (28) and (29) are Rayleigh surface wave secular equations for stress-free, chemical potential, thermally insulated and isothermal boundaries of transversely isotropic thermoelastic diffusive half-space, respectively.

If we write

$$c^{-1} = v^{-1} + i\omega^{-1} F\,, \tag{30}$$

then $\xi = E + iF$, where $E = \frac{\omega}{v}$, v and F are real quantities. The roots of characteristic equation (22) are, in general, complex, i.e., $m_p = p_p + i q_p$, so that the exponent in the plane wave solutions (23) becomes

$$iE\,(x_1 - m_p^{\mathrm{I}} x_3 - vt) - E\left(\frac{F}{E} x_1 + m_p^{\mathrm{R}} x_3\right), \tag{31}$$

where

$$m_p^{\mathrm{R}} = p_p - q_p \frac{F}{E}\,, \quad m_p^{\mathrm{I}} = q_p + p_p \frac{F}{E}\,. \tag{32}$$

This shows that v is the propagation velocity and F is the attenuation coefficient of the wave. Upon using representation (30) in secular equations (28) and (29), one can obtain the values of v and F. The group velocity can be calculated as $U = \frac{d\omega}{dE}$.

Surface Displacements, Temperature Change, and Concentration. The surface displacements, temperature change, and concentration at the surface $x_3 = 0$ in the case of stress-free, chemical potential, thermally insulated or isothermal boundaries of the half-space are

$$(u_{1s}, u_{3s}, T_s, C_s) = (G^*, H^*, I^*, J^*) \, A \exp\,(iE\,(x_1 - vt)] \,, \tag{33}$$

where

$$A = A_1 \exp\,(-Fx_1)\,, \quad G^* = (D_1 - D_2 + D_3 - D_4)/D_1\,, \quad H^* = (n_{11}D_1 - n_{12}D_2 + n_{13}D_3 - n_{14}D_4)/D_1\,,$$

$$I^* = (n_{21}D_1 - n_{22}D_2 + n_{23}D_3 - n_{24}D_4)/D_1\,, \quad J^* = (n_{31}D_1 - n_{32}D_2 + n_{33}D_3 - n_{34}D_4)/D_1\,. \tag{34}$$

Paths of Surface Particles. We now discuss the motion of the surface $x_3 = 0$ particles of the modified Rayleigh waves. It is observed that when thermodiffusive mechanical coupling is operative, the amplitude and velocity of the waves are no longer real, as can be seen from Eq. (33). This means that the wave is damped and the phase difference exists between the functions u_1 and u_3. Therefore on the surface $x_3 = 0$ we have

$$u_{1s} = XA \exp\,\{-i\,(\alpha - q)\}\,, \quad u_{3s} = YA \exp\,\{-i\,(\beta - q)\}\,, \tag{35}$$

where $G^* = X \exp\,(-i\alpha)$, $H^* = Y \exp\,(i\beta)$, $q = E(x_1 - vt)$. Using the Euler representation of complex numbers and simplifying, we obtain from Eq. (35) (retaining only real parts)

$$u_{1s} = XA \cos\,(\alpha - q)\,, \quad u_{3s} = YA \cos\,(\beta - q)\,. \tag{36}$$

Eliminating q from Eq. (36), we get

$$\frac{u_{1s}^2}{X^2} + \frac{u_{3s}^2}{Y^2} - 2\,\frac{u_{1s}u_{3s}}{XY} \cos\,(\alpha - \beta) = A^2 \sin^2\,(\alpha - \beta)\,. \tag{37}$$

Since

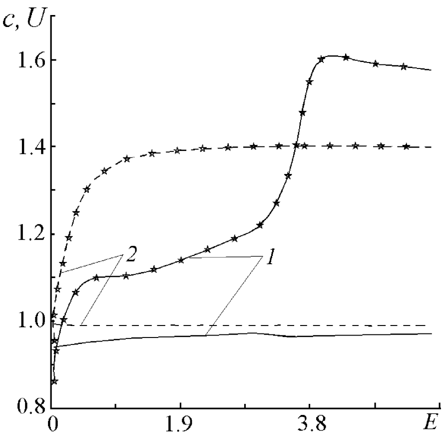
$$\frac{\cos^2\,(\alpha - \beta)}{X^2Y^2} - \frac{1}{X^2Y^2} = -\frac{\sin^2\,(\alpha - \beta)}{X^2Y^2} < 0\,,$$

Eq. (37) represents an ellipse with semimajor axis X^*, semiminor axis Y^*, and eccentricity e which are given by

$$X^{*2} = \frac{2A^2X^2Y^2 \sin^2\,(\alpha - \beta)}{X^2 + Y^2 - [(X^2 - Y^2)^2 + 4X^2Y^2 \cos^2\,(\alpha - \beta)]^{1/2}}\,, \tag{38}$$

$$Y^{*2} = \frac{2A^2X^2Y^2 \sin^2\,(\alpha - \beta)}{X^2 + Y^2 + [(X^2 - Y^2)^2 + 4X^2Y^2 \cos^2\,(\alpha - \beta)]^{1/2}}\,, \tag{39}$$

$$e^2 = \frac{2\,[(X^2 - Y^2)^2 + 4X^2Y^2 \cos^2\,(\alpha - \beta)]^{1/2}}{X^2 + Y^2 + [(X^2 - Y^2)^2 + 4X^2Y^2 \cos^2\,(\alpha - \beta)]^{1/2}}\,. \tag{40}$$



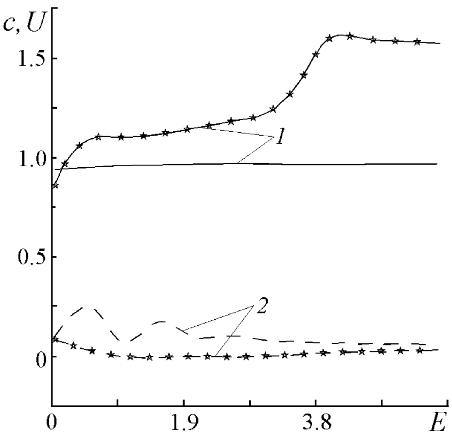


Fig. 1. Variation of the group velocity (curves with asterisks) and the phase velocity versus the wave number for the TID (1) and ID (2) cases.

Fig. 2. Variation of the group velocity (curves with asterisks) and the phase velocity versus the wave number for the TID (1) and TI (2) cases.

If δ^* is the inclination of the major axis to the wave normal,

$$\tan(2\delta^*) = \frac{2XY\cos(\alpha-\beta)}{Y^2-X^2}. \tag{41}$$

Thus, it follows that the surface particles travel elliptical paths given by Eq. (37) in the vertical planes parallel to the direction of wave propagation. The values of the semiaxes depend upon $A = A_1 \exp(-Fx_1)$ and hence increase or decrease exponentially. It is clear that the decay of elliptical paths of surface particles depends on the attenuation coefficient F. For $\alpha - \beta = \frac{\pi}{2}$, both X^* and Y^* have the same signs and therefore the surface particles cover elliptical paths. Moreover, the particle paths degenerate into straight lines, when $\alpha = \beta$, that is, when there is no phase difference between the functions u_{1s} and u_{3s}.

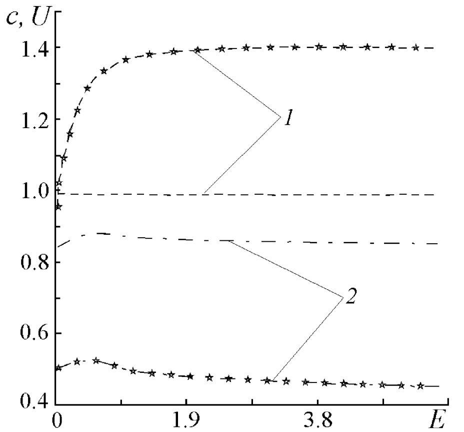
From the above equations (28) and (29), we obtained the equations for *special cases*, which are not given here. For example, in the absence of diffusion or thermal effects, we obtained frequency equations that correspond closely with the relevant equations presented in [23, 24].

Numerical Results and Discussion. For numerical computations, we take the following values of the relevant parameters for transversely isotropic generalized thermoelastic diffusive solid: c_{11} = 16.78·10^{10} kg·m^{-1}·sec^2, c_{12} = 8.76·10^{10} kg·m^{-1}·sec^2, c_{13} = 10^{11} kg·m^{-1}·sec^2, c_{33} = 16.5·10^{10} kg·m^{-1}·sec^2, c_{44} = 5.06·10^{10} kg·m^{-1}·sec^2, T_0 = 0.293·10^3 K, C_E = 0.5031·10^3 J·kg^{-1}·K^{-1}, α_1 = 1.98·10^{-5} K^{-1}, α_3 = 2·10^{-5} K^{-1}, α_{1c} = 2.1·10^{-4} m^3·kg^{-1}, α_{3c} = 2·10^{-4} m^3·kg^{-1}, a = 2·10^4 m^2·sec^{-2}·K^{-1}, b = 10^6 kg·m^5·sec^{-2}, $\tau_0 = 0.03$ sec, $\tau^0 = 0.01$ sec, α_1^* = 1.05·10^{-8} kg·m^{-3}·sec, α_3^* = 10^{-8} kg·m^{-3}·sec, ρ = 8.954·10^3 kg·m^{-3}, K_1 = 0.4·10^3 W·m^{-1}·K^{-1}, K_3 = 0.39·10^3 W·m^{-1}·K^{-1}.

Following Sherief and Saleh [13], for comparison with the isotropic generalized thermoelastic diffusive solid we take the values of relevant parameters for the copper material as T_0 = 0.293·10^3 K, C_E = 0.3831·10^3 J·kg^{-1}·K^{-1}, ρ = 8.954·10^3 kg·m^{-3}, $\tau_0 = 0.03$ sec, $\tau^0 = 0.01$ sec, a = 1.2·10^4 m^2·sec^{-2}·K^{-1}, b = 9·10^5 kg·m^5·sec^{-2}, λ = 7.76·10^{10} kg·m^{-1}·sec^2, μ = 3.86·10^{10} kg·m^{-1}·sec^2, K = 0.386·103 W·m^{-1}·K^{-1}, α = 1.78·10^{-5} K^{-1}, α_c = 2.5·10^{-4} m^3·kg^{-1}, D = 0.85·10^{-8} kg·m^{-3}·sec.

All the obtained characteristics are presented in the figures as functions of the wave number E. The variations of the phase and group velocities for transversely isotropic thermoelastic diffusion (TID), isotropic thermoelastic diffusion (ID), and the corresponding cases without diffusion (TI and I) are plotted in Figs. 1–3.

From Fig. 1 it is seen that for TID the phase velocity first increases slightly and then becomes dispersionless, whereas the group velocity first rises sharply and later begins to decrease slowly. As to the ID case, the phase velocity remains constant and the group velocity first increases sharply, but then becomes constant. The values of the group



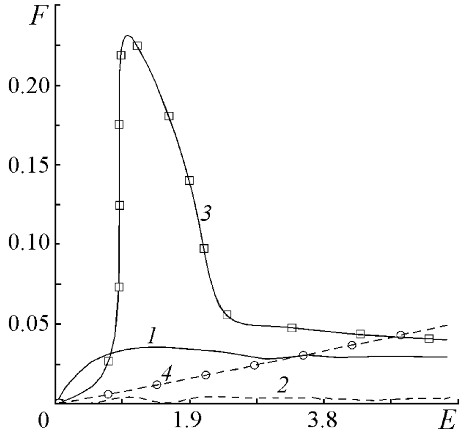
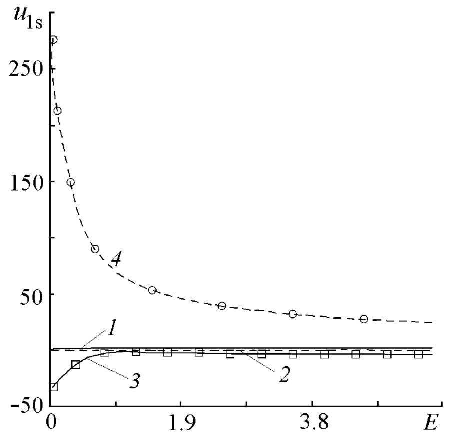


Fig. 3. Variation of the group velocity (curves with asterisks) and the phase velocity versus the wave number for the ID (1) and I (2) cases.

Fig. 4. Variation of the attenuation coefficient versus the wave number for the TID (1), ID (2), TI (3), and I (4) cases. The values for ID are multiplied by 10 and those for TI and I are divided by 100.



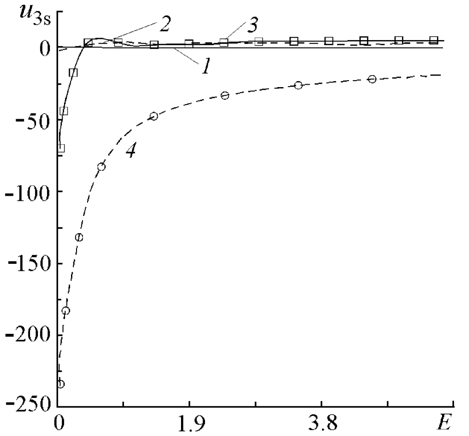


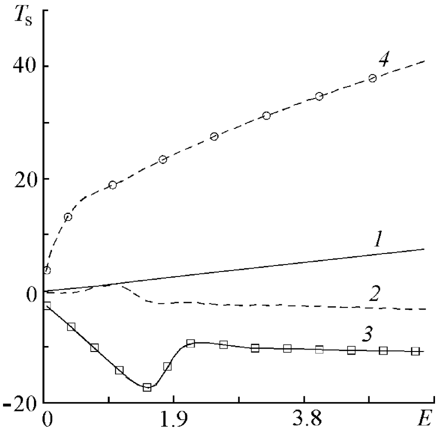
Fig. 5. Variation of the horizontal surface displacement versus the wave number for the TID (1), ID (2), TI (3), and I (4) cases.

Fig. 6. Variation of the vertical surface displacement versus the wave number for the TID (1), ID (2), TI (3), and I (4) cases.

velocity are higher than those of the phase velocity in the both mentioned cases (except the range of low values of *E*). The values of the phase velocity for ID are higher than for TID, whereas initially the group velocity corresponding to ID is higher than that for TID, but further the reverse interrelation takes place. Figure 2 shows that initially the phase velocity for the TI case fluctuates, but then it decreases, whereas the group velocity decreases slowly and then remains constant. Both velocities for TI are lower than those for TID. Similarly, it is seen from Fig. 3 that both velocities for I are lower as compared to those for ID.

The behavior of the attenuation coefficient is shown in Fig. 4. The attenuation coefficient in the TID and ID cases increases slowly. As the wave number increases, there is a larger increase in the values of the attenuation coefficient corresponding to TID as compared to the one for ID. For TI, the attenuation coefficient initially increases sharply, then slumps, and finally becomes constant, whereas for I the coefficient rises steeply.

Figure 5 shows variations of the horizontal surface displacement. This displacement remains nearly constant in the TID and ID cases. The displacement for TI increases and then remains constant, whereas for I it decreases continuously. It is seen that the values in the case of TID are higher as compared to the TI case, and the reverse interrelation is noted in the ID and I cases. Variations of the vertical surface displacement are presented in Fig. 6. The



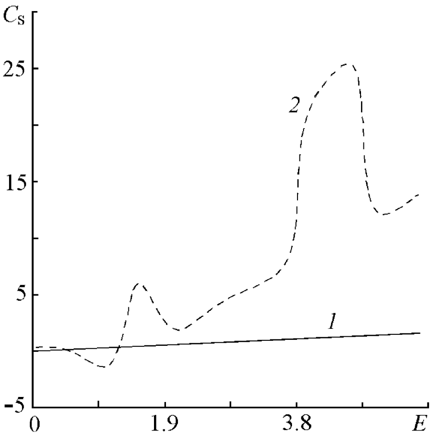


Fig. 7. Variation of the surface temperature change versus the wave number for the TID (1), ID (2), TI (3), and I (4) cases.

Fig. 8. Variation of the surface concentration versus the wave number for the TID (1) and ID (2) cases.

values of this displacement for TID are constant. In the ID case they oscillate and slightly exceed those for TID. The displacement in the TI and I cases increases and then becomes constant. Here, the ID values are higher in comparison with the I ones.

Figure 7 presents graphs of the surface temperature change. In the case of TID the values of the temperature change increase linearly, and in the ID case they decrease and then become constant. For TI, the temperature change initially fluctuates and then also becomes constant, whereas for the I case it increases sharply. The values corresponding to TID are higher as compared to the TI case. The reverse interrelation is noted in the ID and I cases. The variations of the surface concentration are shown in Fig. 8. It is seen that the results for ID are higher in comparison with those for TID.

Conclusions. The propagation of Rayleigh waves in a homogeneous, transversely isotropic, thermoelastic diffusive half-space subjected to stress-free, thermally insulated/isothermal, and chemical potential boundary conditions has been studied. Secular equations for surface wave propagation have been derived. Appreciable transverse isotropy and diffusion effects on the phase and group velocities, attenuation coefficient, surface displacements, temperature change, and concentration have been observed. The results for all the cases considered have been compared. It has been shown that the group velocity remains higher as compared to the phase velocity in the TID and ID cases; both velocities corresponding to TI are lower than those for TID; both velocities for the I case remain lower than the ones for ID.

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NOTATION

a, coefficient describing the measure of thermoelastic diffusion effects; a_{ij}, components of the tensor of thermal modules; b, coefficient describing the measure of diffusion effects; b_{ij}, components of the tensor of diffusion modules; c, dimensionless phase velocity; C, concentration; C_E, specific heat at constant strain; c_{ijkm}, elastic parameters; e_{ij}, components of the strain tensor; E, wave number; F, attenuation coefficient; F_i, external force per unit mass; h, surface heat transfer coefficient; K_{ij}, components of thermal conductivity; M, strength of heat per unit mass; N, strength of mass diffusion source per unit mass; P, chemical potential per unit mass; $\mathbf{q}$, heat flux; S, entropy per unit mass; T, temperature; T_0, reference temperature; u_i, component of displacement vector; U, group velocity; v, phase velocity; v_1, longitudinal wave velocity; x_i, coordinate; α_1, coefficient of linear thermal expansion; α_{1c}, coefficient of diffusion expansion; α^*_{ij}, diffusion parameters; η, flow of diffusion mass; ξ, complex wave number; ρ, density at constant strain; σ_{ij}, components of the stress tensor; τ^0, diffusion relaxation time; τ_0, thermal relaxation time; ϕ, angle of rotation; ω, frequency; ω^*_1, characteristic frequency of the medium. Subscripts and superscripts: I, imaginary; R, real; s, surface.

REFERENCES

1. H. W. Lord and Y. Shulman, A generalized dynamical theory of thermoelasticity, *J. Mech. Phys. Solids*, **15**, 299–309 (1967).
2. A. E. Green and K. A. Lindsay, Thermoelasticity, *J. Elasticity*, **2**, 1–7 (1972).
3. R. S. Dhaliwal and H. Sherief, Generalized thermoelasticity for anisotropic media, *Quart. Appl. Math.*, **33**, 1–8 (1980).
4. D. S. Chanderashekhariah, Thermoelasticity with second sound: a review, *Appl. Mech. Rev.*, **39**, 355–376 (1986).
5. R. B. Hetnarski and J. Ignaczak, Generalized thermoelasticity, *J. Thermal Stresses*, **22**, 451–476 (1999).
6. W. Nowacki, Dynamical problems of thermodiffusion in solids. Part I, *Bull. Pol. Acad. Sci., Ser. Sci. Tech.*, **22**, 55–64 (1974).
7. W. Nowacki, Dynamical problems of thermodiffusion in solids. Part II, *Bull. Pol. Acad. Sci., Ser. Sci. Tech.*, **22**, 129–135 (1974).
8. W. Nowacki, Dynamical problems of thermodiffusion in solids. Part III, *Bull. Pol. Acad. Sci., Ser. Sci. Tech.*, **22**, 275–276 (1974).
9. W. Nowacki, Dynamical problems of thermodiffusion in solids, *Proc. Vib. Prob.*, **15**, 105–128 (1974).
10. W. Dudziak and S. J. Kowalski, Theory of thermodiffusion for solids, *Int. J. Heat Mass Transfer*, **32**, 2005–2013 (1989).
11. Z. S. Olesiak and Y. A. Pyryev, A coupled quasi-stationary problem of thermodiffusion for an elastic cylinder, *Int. J. Eng. Sci.*, **33**, 773–780 (1995).
12. H. H. Sherief, H. Saleh, and F. Hamza, The theory of generalized thermoelastic diffusion, *Int. J. Eng. Sci.*, **42**, 591–608 (2004).
13. H. H. Sherief and H. Saleh, A half-space problem in the theory of generalized thermoelastic diffusion, *Int. J. Solids Struct.*, **42,** 4484–4493 (2005).
14. B. Singh, Reflection of P and SV waves from free surface of an elastic solid with generalized thermodiffusion, *J. Earth Syst. Sci.*, **114**, No. 2, 159–168 (2005).
15. B. Singh, Reflection of SV waves from free surface of an elastic solid in generalized thermodiffusion, *J. Sound Vib.*, **291**, Nos. 3–5, 764–778 (2006).
16. M. Aouadi, Variable electrical and thermal conductivity in the theory of generalized thermodiffusion, ZAMP, **57**, No. 2, 350–366 (2006).
17. M. Aouadi, A generalized thermoelastic diffusion problem for an infinitely long solid cylinder, *Int. J. Math. Math. Sci.*, 1–15 (2006).
18. M. Aouadi, A problem for an infinite elastic body with a spherical cavity in the theory of generalized thermoelastic diffusion, *Int. J. Solids Struct.*, **44**, 5711–5722 (2007).
19. M. Aouadi, Uniqueness and reciprocity theorems in the theory of generalized thermoelastic diffusion, *J. Thermal Stresses*, **30**, 665–678 (2007).
20. M. Aouadi, Generalized theory of thermoelastic diffusion for anisotropic media, *J. Thermal Stresses*, **31**, 270–285 (2008).
21. J. N. Sharma, Generalized thermoelastic diffusive waves in heat conducting materials, *J. Sound Vib.*, **301**, 979–993 (2007).
22. J. N. Sharma, Y. D. Sharma, and P. K. Sharma, On the propagation of elastothermodiffusive surface waves in heat conducting materials, *J. Sound Vib.*, **315**, No. 4, 927–938 (2008).
23. J. N. Sharma, M. Pal, and D. Chand, Propagation characteristics of Rayleigh waves in transversely isotropic piezothermoelastic materials, *J. Sound Vib.*, **284**, 227–248 (2005).
24. W. M. Ewing, W. S. Jardetzky, and F. Press, *Elastic Layers in Layered Med*ia, McGraw-Hill, New York, Toronto, London (1957).

Appendix

$$A^* = \frac{f_1 g_1 + f_2 g_2 - f_3 g_3 - r_7 g_7}{f_2 g_1}, \quad B^* = \frac{f_1 g_2 + f_2 g_4 - f_3 g_5 + r_1 g_6 - r_6 g_7 - r_7 g_{11}}{f_2 g_1},$$

$$C^* = \frac{f_1 g_4 + f_2 g_8 - f_3 g_9 + r_1 g_{10} - r_6 g_{11} + r_7 g_{13}}{f_2 g_1}, \quad D^* = \frac{f_1 g_8 - r_1 g_{12} + r_6 g_{13}}{f_2 g_1},$$

$$\delta_1 = \frac{c_{44}}{c_{11}}, \quad \delta_2 = \frac{c_{13} + c_{44}}{c_{11}}, \quad \delta_4 = \frac{c_{33}}{c_{11}}, \quad \varepsilon_1 = \frac{a_3}{a_1}, \quad \varepsilon_2 = \frac{b_3}{b_1}, \quad \varepsilon_3 = \frac{K_3}{K_1}, \quad \zeta_1 = \frac{a T_0 v_1^2 a_1}{\overset{*}{\omega}_1 K_1 b_1}, \quad \zeta_2 = \frac{a_1^2 T_0}{\rho K_1 \overset{*}{\omega}_1},$$

$$q_1^* = \frac{\overset{*}{\alpha}_1 \overset{*}{\omega}_1 b_1^2}{\rho v_1^4}, \quad q_2^* = \frac{\overset{*}{\alpha}_3 \overset{*}{\omega}_1 b_1 b_3}{\rho v_1^4}, \quad q_3^* = \frac{\overset{*}{\alpha}_1 \overset{*}{\omega}_1 b_1 a}{a_1 v_1^2}, \quad q_4^* = \frac{\overset{*}{\alpha}_3 \overset{*}{\omega}_1 b_1 a}{a_1 v_1^2}, \quad q_5^* = \frac{\overset{*}{\alpha}_1 \overset{*}{\omega}_1 b}{v_1^2}, \quad q_6^* = \frac{\overset{*}{\alpha}_3 \overset{*}{\omega}_1 b}{v_1^2},$$

$$q_7^* = \frac{\overset{*}{\alpha}_3 \overset{*}{\omega}_1 b_1^2}{\rho v_1^4}, \quad q_8^* = \frac{\overset{*}{\alpha}_1 \overset{*}{\omega}_1 b_1 b_3}{\rho v_1^4} \tau_{10} = \tau_0 + i/\omega, \quad \tau_{20} = \tau^0 + i/\omega,$$

$$g_1 = f_6 l_3 - r_8 l_7, \quad g_2 = f_5 l_3 + f_6 l_2 - r_2 l_5 - r_9 l_7 + r_8 l_6, \quad g_3 = f_3 l_3 - r_8 j_2, \quad g_4 = f_5 l_2 + f_6 l_1 - r_2 l_4 + r_9 l_6,$$

$$g_5 = f_3 l_2 - r_2 l_9 + r_8 j_1 - r_9 j_2, \quad g_6 = f_3 l_5 - f_6 l_9 + r_8 j_3, \quad g_7 = f_6 j_2 - f_3 l_7, \quad g_8 = f_5 l_1,$$

$$g_9 = f_3 l_1 - r_2 l_8 + r_9 j_1, \quad g_{10} = f_3 l_4 - f_5 l_9 - f_6 l_8 + r_9 j_3, \quad g_{11} = f_3 l_6 + f_5 j_2 - f_6 j_1 + r_2 j_3, \quad g_{12} = f_5 l_8, \quad g_{13} = f_5 j_1,$$

$$f_1 = \xi^2 (1 - c^2), \quad f_2 = \delta_1 \xi^2, \quad f_3 = \delta_2 \xi^2, \quad f_4 = i\xi, \quad f_5 = \xi^2 (\delta_1 - c^2), \quad f_6 = \delta_4 c^2, \quad f_7 = i\xi\varepsilon_1, \quad f_8 = i\xi\varepsilon_2,$$

$$r_1 = i\omega^2 \xi \zeta_2 \tau_{10}, \quad r_2 = i\omega^2 \xi \zeta_2 \varepsilon_1 \tau_{10}, \quad r_3 = \xi^2 (c^2 \tau_{10} - 1), \quad r_4 = -\varepsilon_3 \xi^2, \quad r_5 = \omega^2 \zeta_1 \tau_{10},$$

$$r_6 = i q_1^* \xi^2, \quad r_7 = i q_7^* \xi^2, \quad r_8 = i q_2^* \xi^2, \quad r_9 = i q_8^* \xi^2,$$

$$h_1 = q_3^* \xi, \quad h_2 = q_4^* \xi, \quad h_3 = -q_5^* \xi + \omega c \tau_{20}, \quad h_4 = -q_6^* \xi,$$

$$l_1 = r_3 h_3 - r_5 h_1, \quad l_2 = h_4 r_3 + r_4 h_3 - h_2 r_5, \quad l_3 = r_4 h_4, \quad l_4 = f_7 h_3 - f_8 h_1, \quad l_5 = f_7 h_4 - h_2 f_8,$$

$$l_6 = f_7 r_5 - f_8 r_3, \quad l_7 = r_4 f_8, \quad l_8 = f_4 (h_3 - h_1), \quad l_9 = (h_4 - h_2) f_4,$$

$$j_1 = f_4 (r_5 - r_3), \quad j_2 = r_4 f_4, \quad j_3 = f_4 (f_8 - f_7),$$

$$I_1 = r_2 h_4 - r_8 r_5, \quad I_2 = f_6 h_4 - f_8 r_8, \quad I_3 = f_6 r_5 - f_8 r_2, \quad I_4 = r_2 h_3 - r_5 r_9, \quad I_5 = f_6 h_3 + f_5 h_4 - f_8 r_9,$$

$$I_6 = r_6 r_5 - r_1 h_3, \quad I_7 = f_6 r_7 - f_3 r_8, \quad I_8 = h_2 r_2 - r_8 r_3 - r_9 r_4, \quad I_9 = f_6 h_2 - f_7 r_8,$$

$$I_{10} = f_6 r_3 + f_5 r_4 + f_7 r_2, \quad I_{11} = r_2 h_1 - r_9 r_3, \quad I_{12} = f_6 h_1 + f_5 h_2 - f_7 r_9, \quad I_{13} = r_6 r_3 - h_1 r_1,$$

$$g_{14} = f_3 l_1 - r_7 l_7, \quad g_{15} = f_3 l_2 - r_1 l_5 - r_6 l_7 + r_7 l_6,$$

$$g_{16} = f_3 l_3 - r_1 l_4 + r_6 l_6, \quad g_{17} = f_3 I_1 - r_1 I_2 + r_7 I_3, \quad g_{18} = f_3 I_4 - r_1 I_5 + r_6 I_3 + r_7 r_5 f_5, \quad g_{19} = f_5 I_6,$$

$$g_{20} = r_4 I_7\,, \quad g_{21} = f_3 I_8 - r_1 I_9 + r_7 I_{10} + r_6 f_6 r_4\,, \quad g_{22} = f_3 I_{11} - r_1 I_{12} + r_6 I_{10} + r_7 f_5 r_3\,, \quad g_{23} = f_5 I_{13}\,,$$

$$n_{1p} = -\frac{m_p^5 g_{14} + m_p^3 g_{15} + m_p g_{16}}{m_p^6 g_1 + m_p^4 g_2 + m_p^2 g_4 + g_8}\,, \quad n_{2p} = \frac{m_p^4 g_{17} + m_p^2 g_{18} + g_{19}}{m_p^6 g_1 + m_p^4 g_2 + m_p^2 g_4 + g_8}\,,$$

$$n_{3p} = -\frac{m_p^6 g_{20} + m_p^4 g_{21} + m_p^2 g_{22} + g_{23}}{m_p^6 g_1 + m_p^4 g_2 + m_p^2 g_4 + g_8}\,, \quad p = 1, 2, 3, 4\,.$$